Is it so bad

that we cannot recognize black swans?

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IAREP/SABE/ICABEEP 2010 Workshop ‘Rationality. Behavior. Experiments’,

Moscow, 1-3 September 2010

Supported by Laboratory DECAN of NRU HSE, NRU HSE Science Foundation (grant № 10-04-0030)
and Magdalene College of University of Cambridge.

Introduction

Analyzing the reasons of financial crises Nicolas Nassim Taleb in his book “The Black Swans” (Penguin Books, London, 2008) states that modern economic models badly describe reality for they are not able to forecast such crises in advance. All extraordinary events, e.g. crises, are named by the author “The Black Swans”. Let us give some quotes.

«What we call here a Black Swan (and capitalize it) is an event with the following three attributes. First, it is an *outlier*, as it lies outside the realm of regular expectations, because nothing in the past can convincingly point to its possibility. Second, it carries an extreme impact. Third, in spite of its outlier status, human nature makes us concoct explanations for its occurrence *after* the fact, making it explainable and predictable.

I stop and summarize the triplet: rarity, extreme impact, and retrospective (though not prospective) predictability.

This combination of low predictability and large impact makes the Black Swan a great puzzle; but that is not yet the core concern of this book. Add to this phenomenon the fact that we tend to act as if it does not exist! I don’t mean just you, your cousin Joey, and me, but almost all “social scientists” who, for over a century, have operated under the false belief that their tools could measure uncertainty. For the applications of the sciences of uncertainty to real-world problems has had ridiculous effects; I have been privileged to see it in finance and economics. Go ask your portfolio manager for his definition of “risk,” and odds are that he will supply you with a *measure* that *excludes* the possibility of the Black Swan–hence one that has no better predictive value for assessing the total risks than astrology (we will see how they dress up the intellectual fraud with mathematics). This problem is endemic in social matters.

The central idea of this book concerns our blindness with respect to randomness, particularly the large deviations: Why do we, scientists or nonscientists, hotshots or regular Joes, tend to see the pennies instead of the dollars? Why do we keep focusing on the minutiae, not the possible significant large events, in spite of the obvious evidence of their huge influence?» (Prologue, pp. XVII-XIX).

Moreover, the author gives the following rather sharp characterization of modern economic knowledge.

«In orthodox economics, rationality became a straitjacket. Platonified economists ignored the fact that people might prefer to do something other than maximize their economic interests. This led to mathematical techniques such as “maximization,” or “optimization,” on which Paul Samuelson built much of his work… It involves complicated mathematics and thus raises a barrier to entry by non-mathematically trained scholars. I would not be the first to say that this optimization set back social science by reducing it from the intellectual and reflective discipline that it was becoming to an attempt at an “exact science.”» (p.184).

And, finally,

«… those who started the game of “formal thinking,” by manufacturing phony premises in order to generate “rigorous” theories, were Paul Samuelson, Merton’s tutor, and, in the United Kingdom, John Hicks. These two wrecked the ideas of John Maynard Keynes, which they tried to formalize (Keynes was interested in uncertainty, and complained about the mind-closing certainties induced by models). Other participants in the formal thinking venture were Kenneth Arrow and Gerard Debreu. All four were Nobeled… All of them can be safely accused of having invented an imaginary world, one that lent itself to their mathematics.» (p.283).

Not being adherents of any particular points of view in the economic community, we have tried to present the processes occurring on the stock exchange in the form of two random processes, one of which occurs frequently (normal mode) and the other – rarely (crisis).

Next, we estimated the average gain with the different probabilities of correct recognition of these processes and used the resulting estimates for the actual processes on the exchange.

Briefly, we've got the following answer: if frequent, regular processes are detected correctly even with a probability slightly higher than $\frac{1}{2}$, it almost always allows to have a positive average gain. This very phenomenon seems underlies the reluctance of people to expect crises all the time and do not try to identify them.

Problem statement

The flow of events of two types – type Q (from *quick*) and type R (from *rare*) – enters the device. Each of them is the simplest, i.e. stationary, ordinary and has no aftereffects. The intensity of the flow of events of type Q is equal to λ , the intensity of the flow of events of R is equal to μ, where $\lambda \gg \mu$ (Q-type events are far more frequent than the *R*-type events).

The problem of the device is to recognize coming event X. If an event Q occurs and device identified it correctly, then it gets a small reward a, if the error occurred, and the event Q has been recognized as the event R, then the device is 'fined' by an amount b. The probabilities of such outcomes are known and equal p_1 and q_1, respectively. Similarly for the events of the type $R -$ correct identification of coming event R will give the value of c, where $c \gg a$, and incorrect recognizing will give loss $-d$, $d \gg b$. After each coming event received values of 'win' / 'loss' are added to the previous amount (Fig. 1).

One of the possible implementations of a random process $Z(t)$, equal to the sum of all values of a random variable X received at the time t is given on Fig. 2.

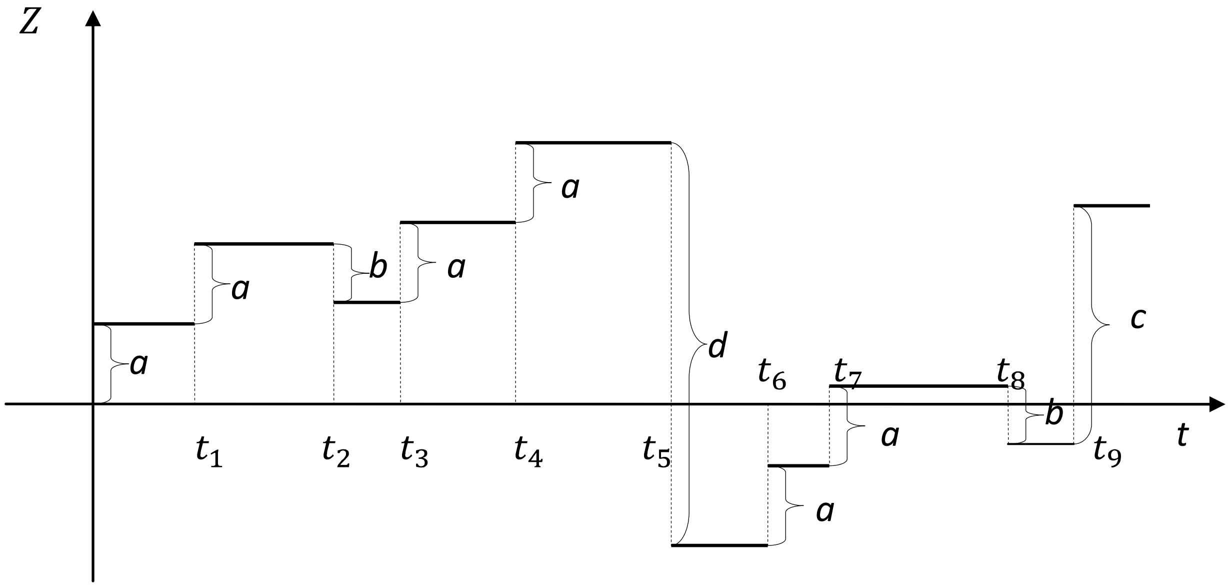

Fig. 1. One of the possible implementations of a random process $Z(t)$

How large on average will be the amount of values received for the time t?

Random value Z of the total sum of the received prizes during the time t is a compound Poisson type variable since the number of terms in the sum $\mathrm{Z} = \sum \mathrm{X_i}$ is also a random variable and depends on the flow of events received by the device.

Expectation of a random variable payoff Z (here W_1 and Y_1 are the random amounts of the gain obtained from the events Q and R, respectively)

$$E(Z) = (\lambda t)E(W_1) + (\mu t)E(Y_1) = \lambda t\big((1 - q_1)a - q_1 b\big) + \mu t\big((1 - q_2)c - q_2 d\big) \quad (1)$$

Analysis of the solution with unknown q_1 and q_2

What conditions should satisfy the values of q_1 and q_2 for the expected value $E(Z)$ to be nonnegative with all other parameters being fixed?

Both q_1 and q_2 are the probabilities of incorrectly recognized events Q and R, so we have to solve the system of inequalities

$$\begin{cases} E(Z) = \left[\lambda\big((1-q_1)a - q_1 b\big) + \mu\big((1-q_2)c - q_2 d\big)\right]t \geq 0, \\ 0 \leq q_1 \leq 1, \\ 0 \leq q_2 \leq 1, \end{cases}$$

when the restrictions on the parameters are $a, b, c, d \geq 0, \lambda, \mu, t > 0, a + b > 0, c + d > 0$ (the last two inequalities mean that both a and b, c and d cannot be equal to zero since the cases with the events with zero losses and gains are not interesting)

$$\begin{cases} q_1\lambda(a+b) + q_2\mu(c+d) \leq \lambda a + \mu c, \\ 0 \leq q_1 \leq 1, \\ 0 \leq q_2 \leq 1. \end{cases}$$

The solution is the range of values q_1 and q_2 defined by the system of inequalities

$$\begin{cases} 0 \le q_1 \le \min\left\{1, \dfrac{\lambda a + \mu c}{\lambda(a+b)}\right\}, \\ \begin{cases} 0 \le q_2 \le 1 \ if \ 0 \le q_1 \le \min\left\{1, \dfrac{\lambda a - \mu d}{\lambda(a+b)}\right\}, \\ 0 \le q_2 \le -\dfrac{\lambda(a+b)}{\mu(c+d)} q_1 + \dfrac{\lambda a + \mu c}{\mu(c+d)} \ if \ \max\left\{0, \dfrac{\lambda a - \mu d}{\lambda(a+b)}\right\} \le q_1 \le \min\left\{1, \dfrac{\lambda a + \mu c}{\lambda(a+b)}\right\}. \end{cases} \end{cases}$$

Analysis of the solution with given q_1 and q_2

Let the values q_1, q_2 be known and predetermined. Then for which values of other parameters the following requirement holds

$$E(Z) = \big((1 - q_1)\lambda a - q_1\lambda b + (1 - q_2)\mu c - q_2\mu d\big) \cdot t \geq 0\ ?$$

Since $t > 0$, $E(Z) \geq 0$ holds when

$$\big((1 - q_1)\lambda a - q_1\lambda b + (1 - q_2)\mu c - q_2\mu d\big) \geq 0$$

or

$$\lambda(a - q_1 a - q_1 b) + \mu(c - q_2 c - q_2 d) \geq 0.$$

3.1 Consider general case $b \neq a, d \neq c$. We find the conditions under which the inequality $\lambda(a - q_1 a - q_1 b) + \mu(c - q_2 c - q_2 d) \geq 0$ holds.

Table 1. The conditions of non-negativity of the total gain

<table>
<tr><td rowspan="5">$E(Y_1)$</td><td colspan="4">$E(W_1)$</td></tr>
<tr><td></td><td>< 0</td><td>$= 0$</td><td>> 0</td></tr>
<tr><td>< 0</td><td>$E(Z) < 0$</td><td>$E(Z) < 0$</td><td>for $E(Z) \geq 0$ required $\frac{\lambda}{\mu} \geq -\frac{E(Y_1)}{E(W_1)}$</td></tr>
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where $E(Y_1) = c - q_2 c - q_2 d$, $E(W_1) = a - q_1 a - q_1 b$.

Let $\lambda = 246, \mu = 4$. If $E(W_1)$ – the expected value of gain from the event of Q-type - is assumed to be equal to 1, then what should be the expected value $E(Y_1)$ of the R-type event?

According to Table 1, the following inequality $E(Y_1) \geq -61{,}5$ holds, i.e., if we have very small expected gain in one unit under frequent observations, we can cover very large losses which happen quite rare.

Consider special case: the gain and loss are the same in both situations, i.e., $b = a, d = c$. Then the condition of nonnegativity of the expectation of the total payoff is $(1 - 2q_1)\lambda a + (1 -$ $2q2\mu c \geq 0$. Dividing by μ and c (both quantities are positive), we obtain

$$(1 - 2q_1)\frac{\lambda}{\mu}\frac{a}{c} \geq (2q_2 - 1).$$

Table 2. The conditions for the non-negativity of the expectation of the gain

		q_1		
		$> 1/2$	$= 1/2$	$< 1/2$
q_2	$> 1/2$	$E(Z) < 0$	$E(Z) < 0$	for $E(Z) \geq 0$ required $\frac{\lambda}{\mu} \geq \frac{2q_2 - 1}{1 - 2q_1} \cdot \frac{c}{a}$
	$= 1/2$	$E(Z) < 0$	$E(Z) = 0$	$E(Z) \geq 0$
	$< 1/2$	for $E(Z) \geq 0$ required $\frac{\lambda}{\mu} \leq \frac{2q_2 - 1}{1 - 2q_1} \cdot \frac{c}{a}$	$E(Z) \geq 0$	$E(Z) \geq 0$

Application to real data

Let the device described above be a stock exchange, and events Q and R describe a 'quiet life' and a 'crisis', respectively. According to the model, Q-events occur more frequently than R that corresponds to the fact that the crises in our lives are fortunately rare.

The event X can be interpreted as a signal received by a broker about the changes in the economy that helps him to decide is the economy in 'a normal mode' or in a crisis.

The values a, b, c, d also have some meaning in such an interpretation. If the event Q occurrs (which means that the economy is stable), and broker correctly recognizes it, then he can get a small income (value a). If the event R will be taken instead of Q, he will lose the amount of $-b$. If the R-event occurred (crisis) and it was not recognized correctly, the broker will lose more (value $-d$). If he could forecast a crisis, he can earn a good deal of money on this, i.e. correct identification of the event of type R gives the broker the value c.

We estimate the parameters of the model, specially the intensity of flows of these events. We will use time series of returns of the stock index S&P 500. The series is stationary as in small samples (about 10-20 points), and for a long period (several years). Periods corresponding to the only regular or the only crises days have been selected to check whether time series are stationary; the Dickey-Fuller Unit root test and analysis of autocorrelation and partial autocorrelation functions were used to control of stationary property.

The time interval has been taken from August 1999 until December 2009. We took the mean of the value of opening and closing as the index value for the day.

We assume the volatility of the index with a sliding interval of 20 days to find when the problems occur in the economy and the recession begins. We took the previous 20 index values S_i for each day and calculate the standard deviation of the sample mean $\bar{S}$ for this sample

$$\sigma_j = \sqrt{\frac{1}{19}\sum_{i=j-20}^{j}(S_i - \bar{S})^2}\,, j = 21,22,\ldots$$

We accept that if the value of volatility is greater than 6%, this means the occurrence of an event of R-type, i.e. a crisis. Figure 3 shows that high values of volatility correspond to the drop of the index.

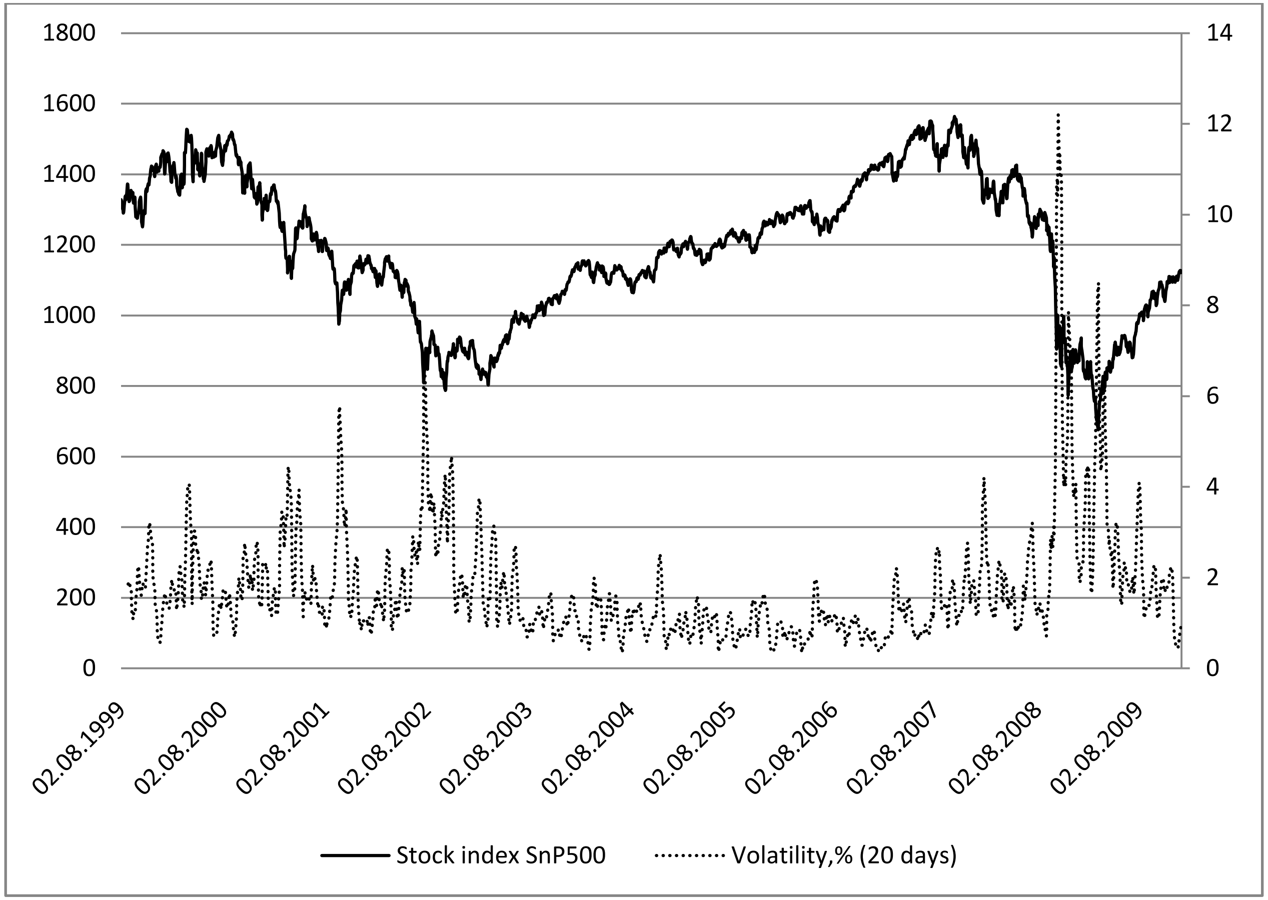

Fig. 2. Stock index S&P500 and its volatility

The threshold 6% was exceeded 42 times of all 2645 (that is 1.6%) for these data (from August 1999 till December 2009). Taking 250 days as an average number of working days per year, we estimate $\mu = 4$ and $\lambda = 246$ for the year. The corresponding estimates are equal to 249 and 1 for the 10% threshold, respectively.

We can find the value of separate 'gain' / 'loss' now. We estimated a and b at points corresponding to the event Q (which is determined by the value of the volatility of the index - it should be less than the threshold value for the event Q). If the index goes up at this moment, it means that the event a was realized, and if it goes down – then $-b$ is observed. The same approach was used for the events of the type R (that we define as the excess of volatility over the threshold value).

We calculated the average in the obtained samples and took them as an estimate of the values of 'gain' / 'loss'.

Such estimates are $a = 7, -b = -7.5, c = 23.5, -d = -26$ in case of 6% threshold value. These values for the threshold value of 10% are shown in Table 3.

Table 3. Estimates for S&P500 index

Stock index S&P500 August 1999 - December 2009 (n =2664 observations)						
Threshold value for volatility	λ	μ	a	$-b$	c	$-d$
6%	246	4	7	-8	24	-26
10%	249	1	7	-8	35	-29

One can see what should be the probability of error in the identification process for the expected gain of broker to be non-negative under these values of parameters. For the 6% threshold the corresponding region in the q_1 - q_2 plane is shown on Fig. 4.

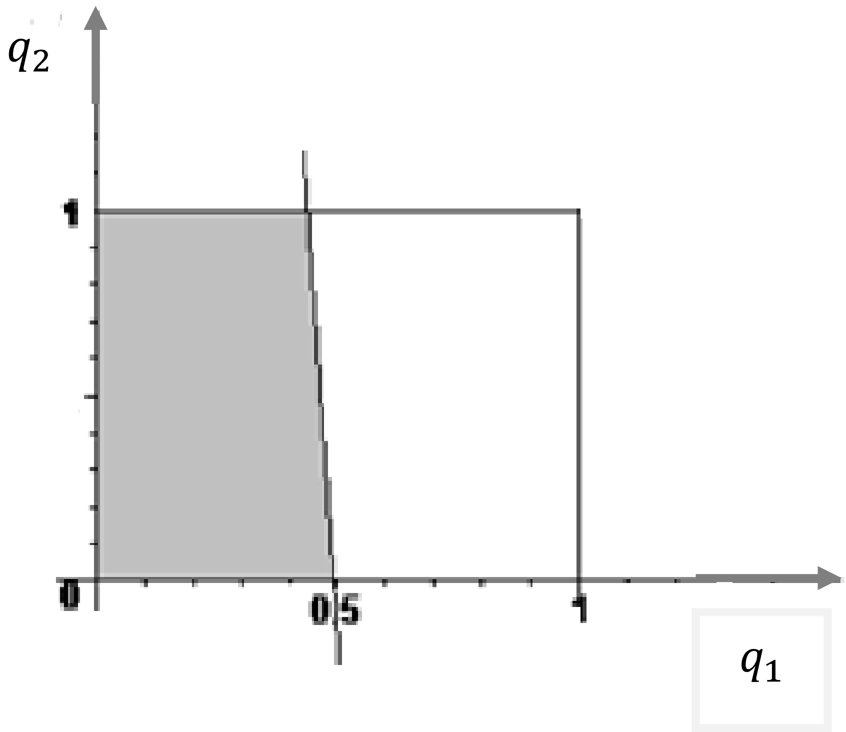

Fig. 3. Area of q_1, q_2, where expected gain is nonnegative.

The evaluation of the indicators for other stock indices

Table 4. Estimates for index CAC 40

Index CAC 40 August 1999-December 2009 (n =2661)						
The threshold for volatility	λ	μ	a	$-b$	c	$-d$
6%	243	7	35	-39	96	-85
10%	249	1	37	-40	133	-170

Table 5. Estimates for index DAX

Index DAX august 1999-december 2009 (n =2651)						
The threshold for the volatility	λ	μ	a	$-b$	c	$-d$
6%	239	11	39	-45	87	-102
10%	249	1	40	-46	106	-158

Table 6. Estimates for index Nikkei 225

Stock index Nikkei 225 august 1999-december 2009 (n =2557)						
The threshold value for volatility	λ	μ	a	$-b$	c	$-d$
6%	245	5	102	-112	232	-299
10%	249	1	103	-114	351	-458

Table 7. Estimates for index Hang Seng

Index Hang Seng august 1999-december 2009 (n =2557)						
The threshold value for volatility	λ	μ	a	$-b$	c	$-d$
6%	241	9	148	-148	390	-481
10%	249	1	155	-159	481	-777

Taking the threshold value as 6%, we compared the estimates obtained for each of the indices.

Table 8. Model estimates for all indices

Estimates for all indices with the threshold 6%						
Index	λ	μ	a	$-b$	c	$-d$
S&P 500	246	4	7	-8	24	-26
CAC 40	243	7	35	-39	96	-85
DAX	239	11	39	-45	87	-102
Nikkei 225	245	5	102	-112	232	-299
Hang Seng	241	9	148	-148	390	-481

A large discrepancy in the estimates of the parameters is observed due to the fact that the changes were taken for estimation of absolute return indices and all values are measured in percentage points. We obtain almost identical values after counting the figures for the relative changes in each index (Table 9).

Table 9. Relative model estimates

Relative estimates for all indices with the threshold 6%				
Index	a	$-b$	c	$-d$
S&P 500	0,6%	-0,6%	2,8%	-2,9%
CAC 40	0,8%	-0,8%	3,0%	-2,5%
DAX	0,8%	-0,9%	2,1%	-2,5%
Nikkei 225	0,8%	-0,9%	2,6%	-3,2%
Hang Seng	0,9%	-0,9%	2,6%	-3,0%

We can see that the relative parameter estimates for the indices are approximately equal – index varies with the scope of 0,8-0,9% in the quiet period and its amplitude increases approximately 4 times in time of crisis.

Remark. It should be noted that these data should reflect the change in the index in percentages compared to the previous day, rather than racing the index for one day. Daily changes may be more significant, for example, in April 17, 2000 the difference between the highest and the lowest index value of Japanese Nikkei 225 amounted to almost 100 points (approximately 10%). However, in April 18 drop was only 3.8% in comparison with the Nikkei in the previous day.

What expected gain will have a broker in our model? If we use the data of S&P 500, choose the horizon of 1 year, and the error probability for events Q and and R being 0,3 and 0,8, respectively, the expected gain is 52%. So, the broker can increase its wealth by 52% a year in average. In case q_1=0.46, q_2=1 the expected gain is positive.

Alternative evaluations of the parameters of the model

Volatility is not the only indicator of market behavior and a signal of crisis.

We carry out the same calculations for estimation of the model parameters, using the index of returns instead of the volatility index

$$r_i = \frac{S_i - S_{i-1}}{S_{i-1}}, i = 2, \ldots 2665.$$

The fall of the index immediately reflected in its return, or more precisely on its amplitude.

Table 10. Estimates of parameters

Index S&P500 August 1999-December 2009 (n =2664)						
The threshold value	λ	μ	a	$-b$	c	$-d$
3%	246	4	7	-7	40	-39
4%	248	2	7	-7	45	-50
5%	249	1	7	-8	47	-51

In addition, the use of average ratings for such a long period of time inevitably oversimplifies the calculations. It is interesting to look at the distribution of crisis events by years (Table 11). The lack of numbers means the absence of volatility 'jumps' more than 6% in a specified period, i.e. 'quiet life' and the absence of shocks in the market.

Table 11. Estimates of model parameters with 6% threshold by years

	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
a	8	11	8	8	5	5	4	5	7	11	8
b	-9	-11	-9	-8	-5	-4	-4	-4	-7	-12	-9
c				17						30	14
d				-25						-30	-15

Conclusion

Defending economics we can say also that R.Schiller could forecast both crises of 2000 and 2005, however, his book did not attract much attention of the scientific community.

It is pointing out by P. Hammond in ‘Adapting to the entirely unpredictable: black swans, fat tails, aberrant events, and hubristic models’, University of Warwick, http://www2.warwick.ac.uk/fac/soc/economics/research/centres/eri/bulletin/2009-10-1/hammond/, 2010 that “…despite its title, Taleb’s book mostly is about how statistical models, especially in finance, should pay more attention to low probability gray swans. It would be much more interesting – though much more challenging – to discuss truly aberrant black swans events to which no probabilities are attached because the model we use does not even contemplate their possibility.”

Instead of analyzing such probabilities, we showed using very simple model that with a small reward for the correct (with probability slightly higher than ½) recognition of the frequent events (and if crisis events are detected with very low probability) the average player's gain will be positive. In other words, players do not need to play more sophisticated games, trying to identify crises events in advance.

This conclusion resembles the logic of precautionary behavior, that prescripts to play the game with almost reliable small wins.